The Intellectual Tradition of Mathematics in Ancient India

R.P. Sharma*

Abstract

The mathematical heritage of ancient Bharat represents one of the most enduring and sophisticated intellectual traditions in human history, tracing its origins to the earliest civilizations of the Indian subcontinent. Archaeological and textual evidence demonstrates a continuous development of mathematical thought extending approximately five millennia, with numerous conceptual innovations that demonstrate remarkable parallels to modern mathematical principles and methodologies. The intrinsic relationship between mathematical inquiry and the cultural fabric of ancient Indian civilization is reflected through an extensive corpus of astronomical texts, architectural treatises, and philosophical works that incorporated advanced mathematical reasoning. This paper undertakes a comprehensive examination of the mathematical innovations originating in ancient Bharat, analyzing their development, applications, and enduring influence on the global mathematical tradition.

Keywords: Sulba Sutra; Vedic mathematics; Vedic proof of Pythagorean Theorem; Invention of zero and negative numbers; Bakhshali Manuscript.

1. Introduction

Albert Einstein once said:

> *"We owe a lot to the Indians, who taught us how to count, without which no worthwhile scientific discoveries could have been made."*

Several great contemporary mathematicians and historians of mathematics, like Einstein, G. P. Halstead, Ginsburg, De Morgan, and Hutton, appreciated India's mathematical heritage and its global influence. These distinguished scholars recognized the epistemological significance of Indian mathematical traditions, with Einstein himself declaring, "We owe a lot to the Indians, who taught us how to count, without which no worthwhile scientific discovery could have been made." Halstead notably acknowledged India as the "cradle of mathematical discovery," emphasizing its pivotal role in shaping numerical systems and computational methodologies. However, many significant contributions of ancient Indian mathematicians to world advancement remain largely unacknowledged, though historical records, archaeological evidence and mathematical achievements from ancient Indian civilization support that many of the Indian innovations from a period starting 5000 years ago are rather parallel to contemporary mathematics (cf. [1, 3, 5, 17-21]). The Indus Valley Civilization (3300 BCE to 1300 BCE) provides compelling evidence suggesting the use of standardized weights and measures, as well as the understanding of geometric principles for town planning and construction. Archaeological excavations at sites such as Mohenjo-Daro and Harappa have revealed sophisticated urban planning based on grid systems, precise measurements using cubits and decimal gradations, and architectural structures demonstrating advanced understanding of geometric proportions. Mathematics and Indians go hand-in-hand, and there are innumerable instances to elaborate this fact, from the conceptual frameworks embedded in ancient religious texts to the quantitative methodologies preserved in astronomical treatises. There is no doubt that India has had a deep impact on the realm of mathematics, with innovations ranging from the formalization of zero to the development of infinitesimal calculus concepts.

The written texts that survive and provide insights into early Indian mathematical thought primarily date from later periods, as much of the earliest mathematical knowledge was transmitted through oral traditions

* Former Professor of Mathematics, Himachal Pradesh University, Summerhill, Shimla-5 (India). Can be reached at rp.math.hpu@gmail.com

(*śruti*) before being codified in written form. This oral transmission system, while posing challenges for precise dating, ensured the preservation and refinement of mathematical concepts across generations. Ancient Indians [10] have made noteworthy innovations that act as a guiding light, even today, demonstrating the temporal transcendence of their mathematical insights. The scholarly competence of Indians and their innovations in various fields are regarded as a source of pride, reflecting a holistic approach to knowledge acquisition where mathematical principles were seamlessly integrated with philosophical, astronomical, and architectural pursuits. A diverse range of fields, from science to mathematics, from astronomy to medicine, from architecture to drama, have been embellished by the academic contribution of Indians. This interdisciplinary synthesis enabled mathematical concepts to emerge from practical needs while simultaneously achieving theoretical sophistication. The contributions made have encouraged the global community in a way that lasts to be at the hub of contemporary scientific development, establishing paradigms that continue to influence modern research methodologies and theoretical frameworks.

The origin of the mathematics that emerged in the Indian subcontinent can be seen around the *Sulba Sutra* period [2], around 1200 BCE to 500 BCE, representing the earliest systematic exposition of geometric principles in ancient India. All that is known as Vedic mathematics is confined to the *Sulba Sutra*s, which primarily address the construction of ritual altars with precise geometrical specifications for religious ceremonies. Indeed, the *Sulba Sutra*s do not contain any proof of the rules that they describe, reflecting the practical orientation of these texts rather than an absence of theoretical understanding. This pragmatic approach suggests that proofs were either transmitted orally or considered self-evident within the ritual context. In [22], some of the rules, such as the method of constructing a square of area equal to a given rectangle, are exact, demonstrating perfect mathematical rigor. Others, such as constructing a square of area equal to that of a given circle (squaring the circle), are approximations that achieved remarkable precision through iterative methods.

The most important of these documents are the Baudhayana *Sulba Sutra,* written about 800 BC, which contains the earliest known statement of the Pythagorean theorem, and the Apastamba *Sulba Sutra,* written about 600 BC, which provides sophisticated algorithms for geometric constructions and numerical approximations. Historians of mathematics studied and wrote about other *Sulba Sutras* of lesser importance, such as the Manava *Sulba Sutra,* written about 750 BC, which focuses on specific construction procedures, and the Katyayana *Sulba Sutra,* written about 200 BC, which demonstrates more advanced computational techniques. The mathematical principles discovered by these early mathematicians laid the foundation for early mathematical thoughts in ancient India and played a significant role in shaping the foundations of mathematics, and are also significant in the present mathematics through concepts such as irrational numbers, recursive algorithms, and geometric transformations. This paper aims to understand the theoretical advancements in fields such as algebra, geometry, number theory, and astronomy, and how these discoveries influenced not only the subcontinent but also had a global impact, tracing the transmission routes through which Indian mathematical knowledge disseminated to the outside world.

2. Golden Age of Indian Mathematics

The following two phases represent a golden age of Indian mathematics [16].

First Phase (1500 BCE- 500 BCE): Vedic Period
Second Phase (400 BCE- 1200 CE): Classical Period

Notable contributions made during these periods are as follows:

- Concepts of symmetry, proportion, and order in the *Atharva Veda*
- Concept and symbol of zero (0), fractions, irrational, and negative numbers
- Decimal number system, based on powers of ten
- The binary numeral system was introduced by Pingla (3rd century BCE) in his work Chandah Shastra. Centuries after its inception, it found application in the field of computer science.
- Value of digits depending on their positions
- Methods for solving quadratic, simultaneous, and linear equations
- Calculation of square roots
- Introduction of trigonometric functions as we know them today
- Calculation of areas and volumes of various geometric shapes
- Divisibility, prime numbers, and solutions of indeterminate equations.

2.1 Vedic Mathematics (1500 BCE- 500 BCE)

The Vedic Mathematics [9] primarily focuses on Vedic mathematical formulas and their practical applications, particularly in simplifying and expediting tedious and cumbersome arithmetical operations. A significant emphasis is placed on the ability to perform these operations mentally, indicating that Vedic Mathematics provides methods that facilitate mental calculation to a considerable extent. In essence, the system aims to streamline and make more efficient various arithmetic tasks through the application of specific Vedic mathematical techniques, for example, the *Parcivartga Sutra* described as a method to quickly and mechanically determine the coefficients and the independent term to find coefficients of equation : the process involves the following steps:

- The difference of the y-coordinates is used as the x-coefficient.
- The difference of the x-coordinates is used as the y-coefficient.

The Sutra suggests substituting the original coordinates of each of the given points into the equation to find the independent term on the right-hand side. For example, with coordinates (9, 17) and (7, -2): Difference of y coordinates is 19 and the difference of x-coordinates is 2. Hence, the equation is

During the Vedic Period Indian scholars and sages composed the Sulba Sutras ([2], First written texts that preserved mathematical knowledge); mainly about construction of different geometrical shapes, using simple tools like ropes and sticks for accurate construction of altars and fire pits for Vedic rituals that demonstrate a sophisticated understanding of divisibility. The word "Sulba" means "cord" or "rule,". The main Sulbha Sutras are as follows:

(i) The Baudhayana Sulba Sutra (800 BC)

> The Pythagorean theorem, claimed by Pythagoras in the 6th century BCE, is already mentioned in the Baudhayana Sulba Sutra. It states that:

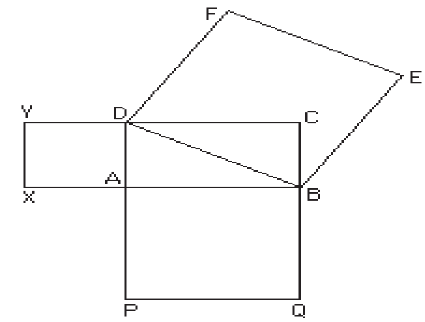
"The rope which is stretched across the diagonal of a square produces an area double the size of the original square. This is the Pythagorean theorem for an isosceles right-angled triangle".

(ii) The Katyayana Sulba Sutra (200 BC)

Though Baudhayana's form [7] is a special form of Pythagoras' theorem, Katyayana Sulba Sutra gives the exact version of the Pythagorean Theorem. It states that:

> "The rope which is stretched along the length of the diagonal of a rectangle produces an area which the vertical and horizontal sides make together".

Katyayana's form and Pythagoras›s form are the same, i.e., the sum of the squares of a right-angled triangle is equal to the sum of the squares of the hypotenuse. It is pertinent to mention that Pythagorean triples already exist in the *Sulba Sutra*, for example: (5, 12, 13), (12, 16, 20),



The Katyayana form of Pythagoras's theorem

(8, 15, 17), (15, 20, 25), (12, 35, 37), (15, 36, 39). Katyayan's *Sulba Sutra* provides some more valuable insights into geometric concepts, including the determination of square roots. His *Sulba Sutra* is considered a testament to the rich mathematical heritage of ancient India, prompting subsequent developments in mathematics throughout the region.

Vedic proof of this theorem (**Pythagorean theorem**) is easier than any other proof available these days. Consider the following figure.

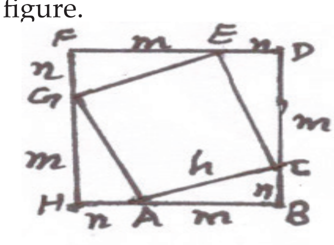


Fig. 1

To prove**:** In right-angled triangle ABC,

$$h^2 = m^2 + n^2 \quad (1)$$

In Fig. 1, the area of the larger square (side = $m + n$) = area of smaller square(side = h) + 4 (area of triangle ABC). So

$(m^2 + n^2) = m^2 + n^2 + 2nm = h^2 + 4/2\ mn$, giving (1) .

(iii) The Manava Sulba Sutra (750 BC)

The main contribution of the Manava's Sulba Sutra [6] is the symbolic notations for mathematical concepts, such as square roots and fractions, which are also used today. Additionally, it contains mathematical and geometric principles. The Manava Sulba Sutra specifically belongs to the Yajurveda tradition and is attributed to the sage Manava (or Manu). It contains mathematical rules and procedures for the construction of sacrificial altars of various shapes, particularly rectangular and square ones. These rules involve geometric principles such as Pythagoras' theorem and techniques for constructing various geometric shapes using cords or ropes.

(iv) The Apastamba Sulba Sutra (600 BC)

Like the Manava Sulba Sutra, it is one of the Sulba Sutras associated with the Vedas, specifically belonging to the Yajurveda tradition. Attributed to the sage Apastamba, the Apastamba Sulba Sutra contains mathematical rules and procedures for constructing various types of sacrificial altars, including rectangular, square, circular, and more complex shapes. Similar to other Sulba Sutras, it provides geometric principles and techniques for accurately constructing these altars using cords or ropes.

The Apastamba Sulba Sutra (cf. [14, 15]), along with other Sulba Sutras, is valuable not only for its mathematical content but also for shedding light on ancient Indian culture, religious practices, and mathematical knowledge. These texts demonstrate the advanced level of geometric understanding and mathematical sophistication present in ancient India and contribute to the history of mathematics. The Sulba Sutras are significant not only for their mathematical content but also for providing insights into ancient Indian culture, religious practices, and the level of mathematical knowledge at that time. They demonstrate the sophisticated understanding of geometry and mathematics possessed by ancient Indian scholars and are considered important texts in the history of mathematics.

2.1. Mathematics during the Classical Period (400 BCE- 1200 CE)

The notable work of this period mentioned in the Aryabhatiya, Brahmasphutasiddhanta, Ganita Sar Sangrah, Lilavati, Vijaganita, and Siddhanta Siromani is listed below to establish the importance of ancient mathematical discoveries of Bharat [23].

(i) Aryabhata (476 CE) Book: Aryabhatiya

Notable Works: Developed mathematical theories for trigonometry, algebra, and arithmetic.

Aryabhata used trigonometry in calculations related to astronomy and planetary motions. He invented zero as a place value notation and decimal number system**.** He used prime numbers in his work and discussed various properties and divisibility rules**.** The Aryabhatiya (cf. [13, 20]) remains an important text in the history of astronomy and mathematics. Aryabhata worked on mathematical concepts, including calculations involving cube roots and square roots. In the Aryabhatiya, Aryabhata presents various mathematical techniques and results. Aryabhata provided various methods for finding square roots. He described a procedure for extracting square roots, and it involved a systematic process. The method involves iterations and is somewhat similar to the process of long division. Aryabhata also discussed cube roots in the Aryabhatiya. Aryabhata's contributions to mathematics extended beyond roots; he made significant advances in algebra, introduced trigonometric concepts, and developed methods for solving indeterminate equations. His work laid the foundation for subsequent developments in Indian mathematics and had a lasting impact on the mathematical traditions in the region. The methods used by Aryabhata were different from modern algorithms, but they were innovative for their time and contributed to the overall progress of mathematical knowledge.

(ii) Brahmagupta (598 CE) Book: Brahmasphutasiddhanta

Notable Works: Introduction of the concept of negative numbers, further development of the concept of zero.

Brahmagupta [23], an ancient Indian mathematician and astronomer from the 6th and 7th centuries BC, made significant contributions to mathematics. Notably, he introduced the concept of zero as a place value and negative numbers, recognizing the need for a numerical placeholder. His use of the symbol "0" was groundbreaking for the development of the decimal number system. Brahmagupta also worked with negative numbers, formulating rules for their operations in his influential work "*Brahmasphutasiddhanta*." This text played a crucial role in shaping the numeral system and algebraic concepts in ancient India and beyond. Additionally, Brahmagupta is credited with providing one of the first systematic and general solutions to quadratic and linear equations using algebraic formulas in the same work. He also developed methods to calculate lunar and solar eclipses.

(iii) Mahaviracharya (815 CE) Book: Ganita Sar Sangrah

Notable Works: Field of algebra, mainly in the development of rational numbers and solutions of quadratic equations.

He established the terminology for concepts such as equilateral and isosceles triangles, rhombus, circle, and semicircle. Also developed the concept of factorial numbers () and their applications in solving combinatorial problems.

(iv) Bhaskaracharya (1114 CE) Book: Lilavati, Vijaganita, Siddhanta Siromani

Notable Works: Significant advances in trigonometry (sine and versine), calculus, and algebra (cf. [11, 12])**.**
He also gave a precise approximation of π (pi) and developed methods to calculate the positions of planets.

3. Bakhshali Manuscript (3rd or 4th century)

This manuscript is written on Birch Bark leaves and contains a wide range of mathematical topics, including arithmetic, algebra, geometry, and quadratic equations. The Bakhshali Manuscript (cf. [8, 16]) is an ancient mathematical manuscript written on birch bark, discovered in 1881 by a farmer in the village of Bakhshali, near Peshawar in modern-day Pakistan. It was found buried in a field along with other ancient artifacts. It's one of the oldest known mathematical texts from the Indian subcontinent, dating back to approximately the 3rd to 4th century CE, though some parts may be even older.

The manuscript eventually came into the possession of a local schoolteacher named Rao Zakharia Kulkarni, who recognized its potential significance and sought to preserve it. Kulkarni sold the manuscript to an antiquities dealer named Anup Chand, who later sold it to the renowned archaeologist Rudolf Hoernle. Rudolf Hoernle, a German-British scholar, recognized the importance of the Bakhshali Manuscript and brought it to the attention of the academic community. He was working in British India at the time and was involved in archaeological and scholarly activities. Hoernle published the first detailed analysis of the manuscript in 1888.

In 1902, Hoernle donated the Bakhshali Manuscript, along with other Indian manuscripts and artifacts, to the Bodleian Library at the University of Oxford. The manuscript has been housed at the Bodleian Library ever since and is considered one of its most valuable treasures. The manuscript contains a wide range of mathematical topics, including arithmetic, algebra, geometry, and methods for solving linear and quadratic equations. One of its notable features is the use of a decimal place-value system, including the use of zero as a placeholder. This makes it one of the earliest known documents to use zero in a mathematical context. The Bakhshali Manuscript provides valuable insights into the mathematical knowledge and practices of ancient India, particularly in the area of computational mathematics. It showcases the sophisticated mathematical understanding of ancient Indian scholars and their contributions to the development of mathematics as a discipline.

Conclusions

The mathematical legacy of ancient Bharat transcended mere theoretical abstraction, embodying a sophisticated epistemological framework that inextricably linked conceptual innovation with practical application. This synthesis of theoretical rigor and empirical utility catalyzed a series of paradigm-shifting discoveries whose reverberations continue to shape contemporary mathematical discourse and methodological approaches.

The conceptualization of zero (**śūnya**) as both a numerical entity and a placeholder during the Vedic period represents a transformative epistemological breakthrough that fundamentally reconstituted the foundations of arithmetic and algebraic systems. This innovation, when coupled with the decimal positional notation system developed by Indian mathematicians, established the structural prerequisites for the universalization of mathematical language and computation. The resultant numerical framework facilitated unprecedented mathematical operations and laid the groundwork for the globalization of mathematical knowledge systems.

The geometrical proofs of the Pythagorean theorem, as articulated in the Sulba Sutras by Baudhayana and Katyayana (circa 800-500 BCE), demonstrate not merely precedence but methodological superiority through their elegant simplicity and accessibility. These proofs, predating the Pythagorean attribution by several centuries, exemplify the pragmatic approach characteristic of ancient Indian mathematics, wherein theoretical constructs emerged organically from practical necessities rather than as purely abstract intellectual exercises.

The mathematical corpus attributed to Aryabhata (476-550 CE) and Brahmagupta (598-668 CE) revolutionized trigonometric analysis through the formalization of the sine function (jyā) and established algorithmic approaches to quadratic equation resolution. Their methodological innovations transcended contemporary frameworks, introducing computational techniques of remarkable sophistication. Bhaskara's subsequent refinements, particularly his work on differential calculus concepts and infinitesimal analysis found in the Lilavati and Bijaganita, demonstrate a sophisticated understanding of limiting processes that presaged modern calculus by several centuries.

The architectural and astronomical applications documented in texts such as the Sulba Sutras reveal the profound integration of mathematical principles within broader cultural and religious frameworks. This contextual embedding ensured not only the preservation but the continuous evolution of mathematical knowledge across generations. The influence of these contributions extended far beyond the geographical boundaries of the Indian subcontinent, propagating through the Islamic world during the medieval period and subsequently permeating European mathematical traditions.

This paper reveals that the mathematical achievements of ancient Bharat represent not isolated innovations but rather components of a comprehensive intellectual ecosystem that approached mathematical inquiry with both pragmatic acuity and theoretical depth. The persistent influence of these contributions on global mathematical development underscores their fundamental significance in the broader narrative of human intellectual achievement, challenging traditional Western-centric historiographies of mathematical progress and establishing ancient India as a crucial locus of mathematical innovation.

References

1. Clark, W. E. (1930). *The Āryabhaṭīya of Āryabhaṭa: An ancient Indian work of mathematics and astronomy*. Chicago: University of Chicago Press.
2. Datta, B. (1932). *The science of the Śulba*. Calcutta.
3. Evans, B. R. (2014). *The development of mathematics throughout the centuries: A brief history in a cultural context*. New York: John Wiley & Sons.
4. Gupta, R. C. (2005). Mystical mathematics of ancient planets. *Indian Journal of History of Science*, *40*, 31–53.
5. Gupta, R. C., & Sarasvati Amma, T. A. (2019). (c. 1920–2000): A great scholar of Indian geometry. *Bhāvanā*, *3*.
6. Gupta, R. C. (1988). New Indian values of π from the Mānava Śulbasūtra. *Centaurus*, *31*(2), 114–125.
7. Gupta, R. C. (1972). Baudhāyana's value of √2. *Mathematics Education*, *6*, B77–B79.
8. Jagadguru Swami Sri Bharati Krishna Tirthaji Maharaj. (n.d.). *Vedic mathematics* (V. S. Agarwal, Ed.). Delhi: Motilal Banarsidass. (Original work published 1965)
9. Joseph, G. G. (2000). *The crest of the peacock: Non-European roots of mathematics* (2nd ed.). Princeton, NJ: Princeton University Press.
10. Joseph, G. G. (2016). *Indian mathematics: Engaging with the world from ancient to modern times*. Hackensack, NJ: World Scientific Publishing.
11. Kak, S. C. (1997). Three old Indian values of π. *Indian Journal of History of Science*, *32*(4), 307–314.
12. Kulkarni, R. P. (1978). The value of π known to Śulbasūtrakāras. *Indian Journal of History of Science*, *13*(1), 32–41.
13. Kumari, G. (1980). Some significant results of algebra of the pre-Āryabhaṭa era. *Mathematics Education (Siwan)*, *14*(1), B5–B13.
14. Mukhopadhyay, A., & Adhikari, M. R. (1997). The concept of cyclic quadrilaterals: Its origin and development in India (from the age of Śulbasūtras to I). *Indian Journal of History of Science*, *32*(1), 53–68.
15. O'Connor, J. J., & Robertson, E. F. (2000, November). *Indian Śulbasūtras*. MacTutor History of Mathematics Archive. https://mathshistory.st-andrews.ac.uk
16. Plofker, K. (2009). *Mathematics in India*. Princeton, NJ: Princeton University Press.
17. Ramanujan, S. (1988). *The lost notebook and other unpublished papers*. Berlin: Springer-Verlag; New Delhi: Narosa Publishing House.
18. Ramasubramanian, K., Hayashi, T., & Montelle, C. (2019). *Bhāskara-prabhā*. *Sources and Studies in the History of Mathematics and Physical Sciences*. Singapore: Springer.
19. Sarasvati Amma, T. A. (1979). *Geometry in ancient and medieval India*. Delhi: Motilal Banarsidass.
20. Srinivasa Rao, A. S. R., & Krantz, S. G. (2021). Dynamical systems: From classical mechanics and astronomy to modern methods. *Journal of the Indian Institute of Science*, *101*, 419–429.
21. Srinivasa Rao, A. S. R. (2006). Ramanujan and religion. *Notices of the American Mathematical Society*, *53*, 1007–1008.
22. Styan, G. P. H., & Chu, K. L. (2013). An illustrated introduction to some old magic squares from India. In *Combinatorial matrix theory and generalized inverses of matrices*. New Delhi: Springer.
23. Sudhākara, D. (1902). *Brahmagupta's Brāhmasphuṭasiddhānta: Edited with Sanskrit commentary*. Benares, India,